PANEVĖŽIO KOLEGIJA ISSN 2029-1280, eISSN 2669-0071. Taikomieji tyrimai studijose ir praktikoje – Applied Research in Studies and Practice, 2022, 18.

A PARAMETER STUDY OF A HYBRID PENDULUM/BALANCING MOBILE ROBOT

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Abstract. Two-wheeled single-axis mobile robots are highly maneuverable. Their radius of gyration is zero. Such robots can be pendulums or balancing (inverted pendulums). A pendulum robot can transform into a balancing robot gaining additional advantages. The influence of the inertia moments and the pendulum mass on the robot's movement and transformation process was investigated in this work. A robot with improperly selected parameters will not be able to transform at all. In other cases, the transformation will take a very long time and during it the robot will travel a long distance. Therefore, it is necessary to evaluate these parameters already during the design of the robot.

Keywords: mobile robot, pendulum, balance, mobile robot dynamics

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INTRODUCTION

Two-wheeled single-axis mobile robots have good maneuverability (Kedrowski, Reinholtz, Abbott & Conner, 2001). They can rotate in place while rolling with zero radius of gyration (Salerno, 2008). These robots need only as much space to turn around as it occupies themselves. From a structural point of view, they are of two types - pendulum and balancing. The parameters of the robots used in the research (geometric sizes, weights, moments of inertia) are presented as constants and are not changed during the research (Haitao, Feng, Xu, Zhang & Fu, 2021).

Pendulum robots have a massive pendulum hanging on the axle of the wheels (Fig. 1, a.), where: 1 - pendulum mass; 2 – link of the pendulum; 3 – wheels; 4 – motor stator; 5 - connection of the motor rotor with the wheel. The mass of the pendulum may consist of electric motors, batteries, other equipment or simply ballast weight (Oryschuk, Salerno, Al-Husseini & Angeles, 2009). Thrust force is created by swinging the pendulum forward or backward from a vertical position. The magnitude of the traction force depends on the pendulum's swing angle and mass. Such robots are stable due to the center of mass below the axis. They don't use energy while standing. The maximum traction force is limited by the length of the pendulum with the mass. The pendulum cannot touch the pavement, therefore it is always shorter than the length of the wheel radius. However, the pendulum can transform into a support enabling the robot to climb stairs (Wu G., Wu L., Wang H., Yang, Wang Z., Zhang, & Shen, 2022).

The mass of the pendulum of balanced robots is above the axis of the wheels (Fig. 1, b.). Such a robot is theoretically stable when the pendulum is bent vertically upwards exactly. However, in this pose the robot must balance, because the pendulum must not tilt to any side. A balancing robot uses energy to maintain balance while standing. But, sooner or later, when the robot's motors are powered off, the pendulum falls down and the robot collapses.

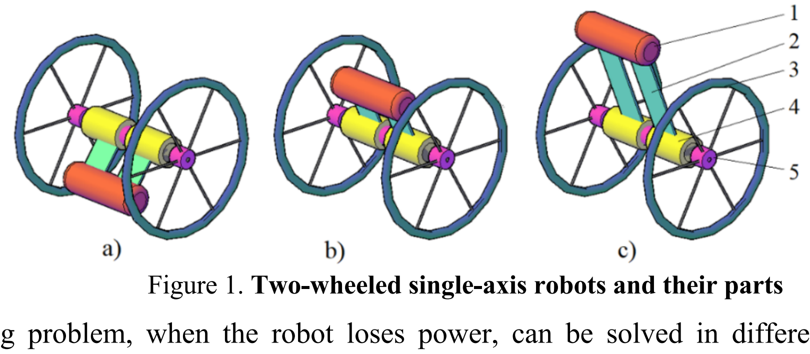


Figure 1. **Two-wheeled single-axis robots and their parts**

The falling problem, when the robot loses power, can be solved in different ways. Additional supports are used (Wahyudi, 2010) to rest the robot on the pavement when the power is turned off. Also auxiliary wheels at the ends of the supports can be used. But in this case the two-wheeled robot temporarily becomes a three-wheeled one and therefore it loses all the advantages of the two-wheeled robot. During non-working hours, special parking spaces can be used where the robot is kept upright or "sits down" (SeongHee & Takayuki, 2007). Usually this problem is not solved at all - the robot just falls down and someone has to put it in an upright state (Kausar, Stol & Patel, 2012). The traction force of the balancing robot depends on the mass of the pendulum also. But in this case, the mass of the pendulum can be raised quite high above the wheel (Fig. 1, c) (Ebrahimi & Mardani, 2015).

The main problem for the balance robots is an impassable obstacle. If the wheel cannot go over the obstacle and rests on it, the robot loses its ability to balance. It falls from inertia in the direction of motion. Therefore balancing robots move relatively slowly on uneven surfaces. Meanwhile, pendulum robots always remain stable when leaning on an obstacle.

The ability of a two-wheeled single-axis robot to move as a pendulum or balance robot would combine the advantages of both types of robots (Duchon, Rodina, Hubinsky, Rau & Kostros, 2016). There are several research of such dual robots, but there the other issues are analyzed (Salerno & Angeles, 2007). For example, what happens if a load is placed on top of a pendulum robot and the center of gravity of the robot appears above the axis of the wheels? Then the robot must immediately start balancing. "Switching" a robot from a pendulum to balance is complicated by requiring more powerful actuators than just moving. During this “switching”, the robot moves from a stable state and requires additional free space to perform this action. In some robots this “switching” from one type to another occurs due to the change in the position of the manipulator and the load, here no separate transformation is even required (Mardany & Ebrahimi, 2015).

The mathematical model of the dynamics of a two-wheel pendulum or balancing mobile robot has many nonlinear elements (Grasser, D'Arrigo, Colombi & Rufer, 2002) and their mathematical models are slightly different. To facilitate the solution of equations, such differential equations are usually simplified by using various methods (An & Li, 2013). Such a simplified mathematical description is valid only for small deviations of the pendulum from the vertical position and cannot be used when the pendulum moves through large angles. This means that the simplified mathematical model of a pendulum robot is not suitable for a balancing robot. If the robot moves on a curvilinear surface, the response of the robot to changes in wheel support in the vertical direction must be included in the mathematical model (Peng, Ruan & Zuo, 2012). Another simplification is used when the armature circuit dynamics of a DC motor are ignored (Grasser, D'Arrigo, Colombi & Rufer, 2002). Ignoring the inductance of the armature circuit is suitable when the robot moves smoothly, without significant disturbances. However, when a pendulum robot turns into a balance, the main factor is the torque created by the current, when the current reaches its maximum values and changes quite quickly.

MATHEMATICAL MODEL OF MOBILE ROBOT

The mathematical model of the dynamics of a mobile robot is intended to study the influence of mechanical parameters on the operation of the robot even before constructing a real robot. In the first stage, the general not simplified mathematical model of the pendulum and balancing robot is designed to study the process of “switching” from one state to another. During “switching”, the robot does not maneuver, so only the planar motion is studied (Kim Y., Kim S. & Kwak, 2005). Here, both motors of the robot work synchronously, the robot can move in the direction of one axis only. An equivalent kinematic representation scheme of one wheel and one electric motor is usually used in the mathematical model (Fig. 2). The stator of the electric motor with mass m_S, is connected to the pendulum mass m_P by a rigid connection of length l (Fig. 2, a). The rotor of the electric motor with mass m_R, is rigidly connected with spokes to the wheel of the robot of mass m_W. The outer radius of the wheel is r_W (Fig. 2, b). All inertial rotating parts rotate around the rotor axis, J_R – the total moment of inertia of the wheel and rotor structure; J_P – the total moment of inertia of the pendulum, connection and stator structure.

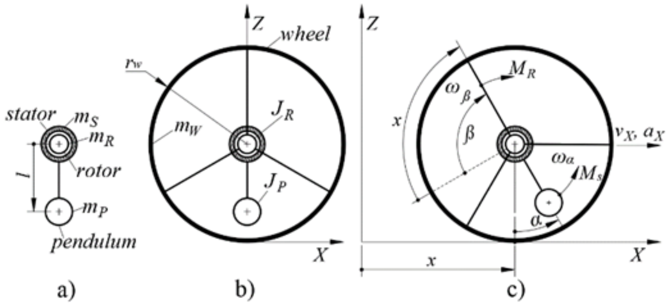


Figure 2. **Planar robot elements and motion directions**

During the movement of the robot, the pendulum with the stator, under the influence of the torque of the motor, turns at an angle α from the vertical position (Fig. 2, c). The same engine torque affects the rotor-wheel system, only in the other direction. The wheel starts rolling from the initial position and rotates through an angle *β*. It is assumed that the wheel rolls without slipping, so the wheel rolls a distance $x=r_w \cdot \beta$.

DC motors with permanent magnets are used to rotate the wheels. Their armature circuit resistance *r*, inductance *L*, generator electromotive force coefficient C_E, electromechanical constant C_M. The stator and rotor of the motor are connected to the moving parts. The stator can rotate at ω_α and the rotor at ω_β angular velocities. The dynamics of such a planar robot are described by the following system of differential equations:

$$\left.\begin{array}{l} L\dfrac{di}{dt} = u - i \cdot r - C_E \cdot (\omega_\beta + \omega_\alpha) \\ J_R\dfrac{d\omega_\beta}{dt} = C_M \cdot i - \omega_\beta \cdot C_2 + m_p \cdot sin\alpha \cdot r_W \cdot (g \cdot cos\alpha + l \cdot \omega_\alpha^2 - a_X \cdot sin\alpha) - r_W \cdot a_X \cdot (m_W + m_S + m_R) \\ J_P\dfrac{d\omega_\alpha}{dt} = C_M \cdot i - \omega_\alpha \cdot D_2 - l \cdot m_p \cdot (g \cdot sin\alpha - a_X \cdot cos\alpha) \\ a_X = r_w \cdot \dfrac{d\omega_\beta}{dt} \end{array}\right\}, \qquad (1)$$

here: *u* – voltage of the armature circuit, *i* – current of the armature circuit, C_2 – coefficient of liquid friction of the rotor system, D_2 – coefficient of liquid friction of the stator system, a_X – linear acceleration of the robot.

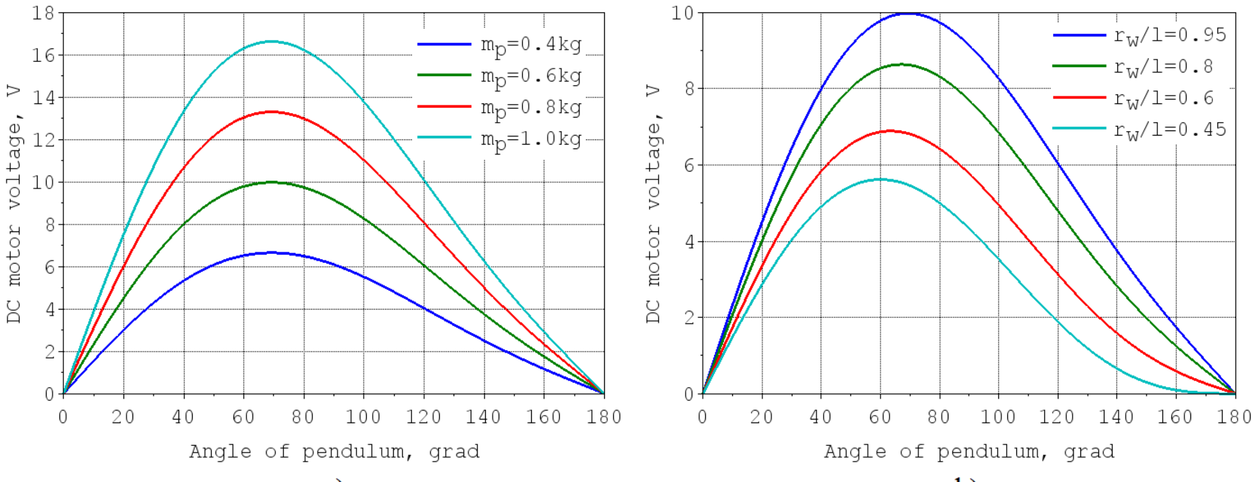
MODELING OF STATIC CHARACTERISTICS

The simulation of the statics and dynamics of the robot is performed by choosing the reference dimensions and parameters of the robot, such as the rolling speed (Herrera-Cordero, Arias-Montiel & Lugo-González, 2018), the time during which the robot transforms from a pendulum to balancing robot. Parameters of the investigated robot: $m_p = 0.6$ kg; $l = 0.3$ m; r_w=0.4 m - radius of the wheel; $m_r = 1.1$ kg. DC motor is C42-L70 Winding 10 type with rated torque 1.8 Nm, terminal voltage 36 V.

The robot's motion is steady when all the derivatives of the variables and the angular velocity of the pendulum are equal to zero. Then, from the system of equations (1), we find the dependence of the motor voltage on the turn angle of the pendulum:

$$u = l \cdot m_P \cdot g \cdot sin\alpha \cdot \left[\frac{r}{C_M} + \frac{C_E}{C_2} \cdot \left(1 + \frac{r_W}{l} \cdot cos\alpha\right)\right]. \qquad (2)$$

For the selected structure and motor, the dependences of the motor voltage on the tilting angle of the pendulum in the range of 0÷180° were calculated for two variants of the design. The first one, is when a pendulum of different mass m_P is used (Fig. 3, a). The second option, is when a pendulum connection of different lengths *l* is used (Fig. 3, b). The ratio of the radius of the wheel and the length of the pendulum was used in the calculation. In both cases, the required motor voltage does not exceed half the terminal voltage.



a) b)

Figure 3. **Dependence of the turn angle of the pendulum on the motor voltage**

Another important characteristic is the steady-state speed v_x of the robot in the horizontal direction. A static equation was created from the system of equations (1), which describes the dependence of the speed v_x of the robot on the turn angle of the pendulum:

$$\omega_\beta = \frac{l \cdot m_p \cdot g \cdot sin\alpha}{C_2}\left(1 + \frac{r_W}{l} \cdot cos\alpha\right). \qquad (3)$$

It is common to analyze the statics or dynamics of pendulum robots using small turn angles of the pendulum. However, equation (3) shows what the speed of the robot will be, when the pendulum of this robot goes up (Figure 4). A pendulum robot can somehow extend the length of the pendulum by lifting the pendulum up. Therefore, the length of the pendulum connection 1.8 times greater than the length of the wheel radius was used in the simulation. Such a pendulum cannot descend, the influence of its turn on the speed is calculated only from a tilt angle of 60 degrees.

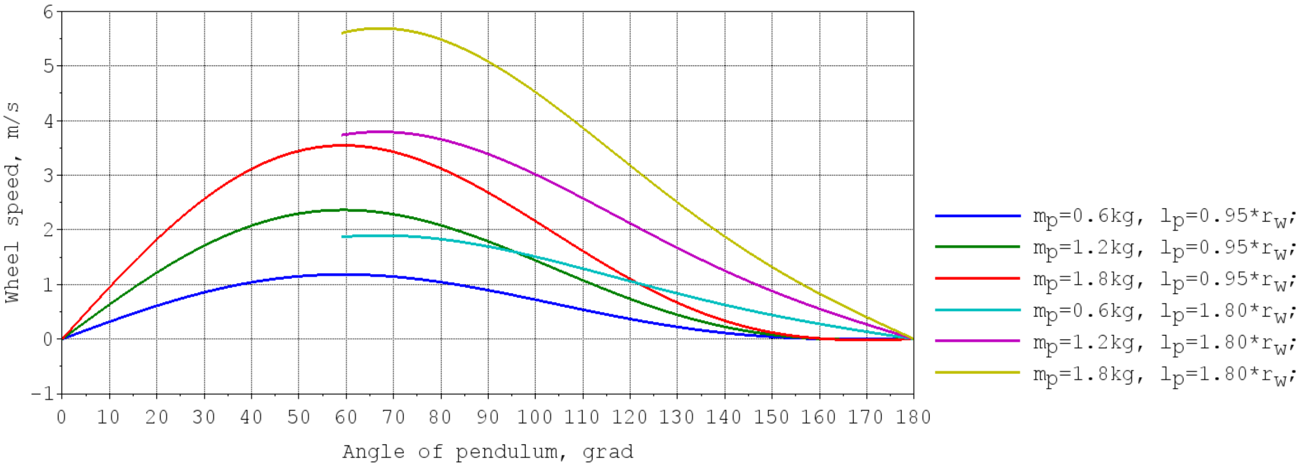


Figure 4. **Dependencies of the robot's movement speed on the turn angle of the pendulum**

Only fluid friction is evaluated in the system of differential equations (1). Evaluation of fluid friction is left for further research with a real robot. However, it can be preliminarily stated, that a pendulum robot of the specified geometric dimensions with a 0.6 kg pendulum can reach a speed of about 1 m/s.

MODELING OF DYNAMIC CHARACTERISTICS

A unique feature of the pendulum/balancing robot is the ability to move the pendulum from the lower position to the upper position. During the transfer process, the maximum available voltage of the robot is applied to the motor and the pendulum moves up in the shortest time. During the process, the maximum possible torque of the motor is applied. As the pendulum rises, the robot itself starts to move and travels the x path until the end of the process.

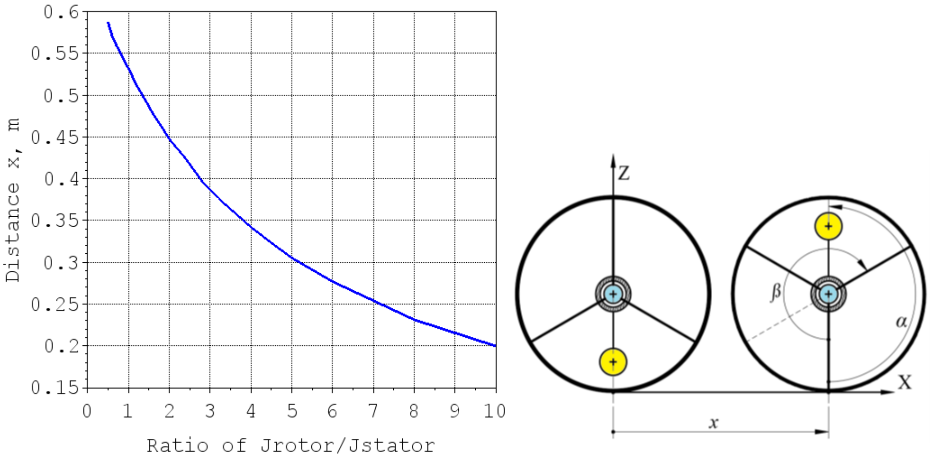


Figure 5. **The distance traveled by the robot during the transformation**

What does the length of this path depend on? The analysis of the solution of differential equations (1) showed that the ratio of moments of inertia of the stator and rotor systems J_R/J_P has the greatest influence on the traveled distance. SCILAB Xcos tools were used for the numerical solution of the equations.

CONCLUSIONS

A mathematical model of the dynamics of a two-wheeled robot was created. The possibility for a pendulum robot to transform into a balancing robot after moving the pendulum upwards has been analyzed. The created equations of the robot's static characteristics allow evaluation of the capabilities of the robot's construction and the selected motors. A study of the dynamics of the transformation from a pendulum to a balancing robot was performed.

1. It was established that if the motor is able to lift the pendulum up, then the robot's movement speed depends only on the swing angle of the pendulum.
2. During the transformation from pendulum to balance, the robot travels a distance, which depends on the ratio of the moments of inertia of the rotor and stator systems.

The used mathematical model evaluates only the influence of fluid friction. The influence of static friction on the movement will be studied after realizing the real construction of the robot.

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